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**Lewis Carroll and Mathematical Ideals of John Allen Paulos:
Review of *Alice's Adventures in Wonderland* (1865) and *Through
the Looking-Glass, and What Alice Found There* (1871)**

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Lewis Carroll and Mathematical Ideals of John Allen Paulos: Review of *Alice's Adventures in Wonderland* (1865) and *Through the Looking-Glass, and What Alice Found There* (1871)

Abstract

At first blush it may seem that linking the acclaimed achievements of John Allen Paulos and the acclaimed achievements of Lewis Carroll (a.k.a. Oxford mathematics don Charles Lutwidge Dodgson) is merely an exercise in free association. Both are prestigious academic mathematicians. Both have an obvious interest in humor. Both have made it to best-seller lists.

That free association, however, is not the issue here. Instead, the issue is whether John Allen Paulos has highlighted basic questions of mathematical literacy and whether the issues that Paulos highlights do not, in fact, reflect mathematical and artistic concerns of Lewis Carroll in writing his immortal classics, *Alice's Adventures in Wonderland* and *Through a Looking Glass*. How can these immortal classics be read—given that they are written in a disguised, symbolic, and literary form—to explain Carroll's wide attraction for adult mathematical and STEM-discipline audiences? And when we have read them so, do Carroll's concerns adumbrate Paulos' insistence on mathematics as a set of mind, a disjunction between that mindset and the mindset of the society around it?

Specifically the intent of this paper is to suggest a consistent reading of Carroll in light of Paulos and to do so in such a way that it suggests important new directions for the discussion of Numeracy (as the opposite of Paulos' Innumeracy) and for the discussion of Quantitative Literacy.

Keywords

Carroll, Paulos, Wonderland, Alice, mathematics

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Cover Page Footnote

Paul Grawe is the Program Director of the Institute for Travesty, Comedy, and Humor Studies (ITCHS.org) and Professor Emeritus of English at Winona State University (Minnesota). His *Comedy in Space, Time, and the Imagination* (Chicago: Nelson-Hall Publishers, 1983) can be found at 593 libraries ranging from the Orlando (Florida) Public Library to the University of Western Australia. In recent years, he and Robin Jaeckle Grawe (Executive Director of ITCHS) have published additional book-length studies on Shakespearean and modern American film comedy (available at the ITCHS website). As a sidelight, Paul is now exploring connections between humor and mathematics/QL in *Numeracy* and the *Journal of Humanistic Mathematics*.

Introduction

I come at Lewis Carroll from a literary perspective, and particularly from the perspective of humor studies. For my purposes here, the humor of Lewis Carroll means overwhelmingly the humor of *Alice's Adventures in Wonderland* and *Through a Looking Glass*. Of the two, it means mainly the latter, *Alice in Wonderland* being largely a happy accident of an Oxford mathematics don whiling away an over-hot afternoon by extending one tale into another before an audience of "little girls." That afternoon of July 4, 1862, evidently "got him going." And eventually that got him into a serious literary effort not to spin a story forward extemporaneously but to devise in *Through a Looking Glass* a work with an over-all, even a grand design, including a signature sense of humor. Carroll would spend significant amounts of time throughout the rest of his life amending, emending, and extrapolating on that endeavor started perhaps by accident but continued in grand design.

Both books of course, from a Humanities point of view are "fantasies." And they both have fantastic humor, that is, humor that derives from fantasy itself. That fantasy has a humorous appeal of its own is not news. But let us look at the special fantasy that humorously initiates *Alice in Wonderland*. This fantasy may be the "happy accident" that propels everything else.

Down the Rabbit Hole with Alice

Alice is idling on the stream bank, increasingly inattentive. Read this as Alice is inattentive to non-humorous mundane goings-on.

And then she sees a White Rabbit, and moreover sees him take out his pocket watch. Read this as humor has entered Alice's world; it has drawn her attention but much more her interest. And ours with hers.

That gets her moving. She follows the Rabbit and finally falls down the Rabbit Hole he has entered. And in a few moments more, she has fallen off a sloping ledge and is in a free-fall descent. There seems no bottom to the pit.

And yet, at some point, she comes to a soft landing in a pile of sticks and leaves.

Everything in *Alice* is premised on this descent into the abyss. And, therefore, all the humor of her subsequent adventures in this volume is humor premised on being *humor of the abyss*, humor of a subterranean world unknown to the pedestrian and terrestrial mundane world above.

As utterly removed and utterly incomprehensible to the world above, Wonderland is a world of nonsense. Or so it is often read. Perhaps “different sense” is more to the point than “nonsense,” and the difference of sense forces a strong persistent incongruity in the humor of *Alice in Wonderland*. As Carroll goes on to make super-abundantly clear, it is also a world of word play. The combination of Word Play and Incongruity (used with capital letters to denote their definitions within a quadrilateral of four sub-forms of Humor of the Mind as we have strongly argued elsewhere¹), makes the humor of Alice “Intellectual.” And thus, the humor of *Alice* is the Intellectual humor of the abyss, or perhaps abysmal Intellectual humor. One way or the other, as Martin Gardner has argued in his introduction to *The Annotated Alice*, “The fact is that Carroll’s nonsense is not nearly as random and pointless as it seems. . .” (Carroll 1990, Gardner *xiv*). We’d argue, his nonsense is “Intellectual other” sense.

The Humor Connection

Let us stop here for the intrusion of a small amount of humor theory. Like a good deal of mathematics, humor has its terrestrial aspect and a very different aspect once we have fallen down the Rabbit Hole of deeper investigation.

At the terrestrial level, humor is all frills, foam, froth and fluff. It is airy nonsense, an appearance that soon dissipates into “just nothing.” As a result, after almost three millennia of “investigation,” since Solomon at least, humor scholarship often has reached virtually no conclusion greater than “Well, that was fun, wasn’t it?”

The Rabbit Hole to be fallen into is pretty much a direct contradiction of the terrestrial: things are humorous not because they are frivolous but rather in proportion to their *importance* to us.

For example, check any nightclub. The humor has a routine sexual content. (This contention was amply supported by speakers at the Oakland, California conference of the International Society for Humor Studies in the general session entitled Berkeley Repertory Roundtable—The Art of Creating Comedy; Mayotte 1999). Why is this? Is it because humor is all foam and fluff and so is sex? We may tell ourselves so in highly inebriated moments.

For something foamy and fluffy, however, sex has been empirically shown to take up an inordinate attention space for adult, responsible, average human beings, most particularly for adult, responsible, average *male* human beings. Check sex jokes with men and women. I think you will find the men far more

¹ For a crash course in humor-of-the-mind personality types, see Grawe and Grawe (1994). For a considerably more developed discussion of humor personality and texture in film and dramatic comedy, see *Comedic Tenor, Comic Vehicle* (Grawe and Grawe 2008) and *Cheshire Smile* (Grawe and Grawe 2014).

interested and far more titillated. They may even read-in sex jokes where none was intended, or at least not admitted.

Well, come on, sex is an unfair example. Hitting below the belt, really!

Okay, how about this at a far remove from sex jokes: Is there empirical *proof* that what is important is what is potentially humorous? The Institute for Travesty, Comedy, and Humor Studies (ITCHS) has been investigating four *forms* of humor—not subject matters of humor—for better than a quarter of a century. In this investigation using the Humor Quotient Test (Grawe, R 1994), we pitted jokes of different forms against each other to gauge the forms' relative power.

In our earliest experiment, we found a then-current Sidney Harris joke about South Sea Islanders meeting in council. The chief announces the result: “So, by a vote of 8 to 2, we have decided to skip the Industrial Revolution completely, and go right into the Electronic Age” (Harris 1989).

You can tell how long ago we did this research. Today, South Sea Islanders have routinely skipped the Industrial Revolution and are talking by cell phones.

But at the time, this was a great example of Incongruity humor. ITCHS had the opportunity to test this joke and an equally funny joke of a different form among a first group of respondents.

It turned out that, like almost any group available, the group we chose had certain personal idiosyncrasies. In this case, 11 of 17 in the group were idiosyncratically people who had had substantial first-hand professional experience with natives in Papua New Guinea. I am proud to say that I recognized that this idiosyncrasy would give us a very false read about the perceived humor in the joke outlined above. In the event, all 11 of the Papua New Guinea-related respondents chose the outlined joke over its opponent. We have never seen anything like such a highly concentrated preference for this joke within a test subgroup in better than 4,000 subsequent respondents to the Humor Quotient Test.

This remarkable skew demonstrates that what concerns us personally and deeply is automatically what is most valued when treated humorously. Humor allows us to move beyond commonplace understanding—it bends the intellectual space like mass bends Einsteinian space. And with that interest and excitement, we are pleased, but more importantly, we are often informed.

And thus back to *Alice in Wonderland* where we have become interested in falling down a Rabbit Hole. Falling out of ordinary experience is a common denominator for the world of the mind. So we can expect that people involved in intellectual pursuits will more likely find the ensuing humor interesting and fun. The evidence is that by the early 21st century, it was intellectuals in the STEM disciplines who still found special fun in Lewis Carroll. As Gardner also remarked, “It is only because adults—scientists and mathematicians in particular—continue to relish the Alice books that they are assured of immortality” (Carroll 1990, Gardner *xiv*). Presumably, that means that the humor

of Lewis Carroll particularly focused on issues of deep interest and relevance to STEM thinking.

Back Down the Rabbit Hole

So, what if the descent after falling down the Rabbit Hole is also the descent into mathematics and into *a mathematical viewpoint*?

Mathematics then is in some ways an accident that happens to good people momentarily distracted into following rabbits—now that would be a funny perspective, wouldn't it?

Down the Rabbit Hole is into a new world, a parallel universe that evidently has been there at least a very long time, perhaps forever, but is entirely unperceived and unappreciated by the uninitiated like the inattentive Alice we first met.

(Incidentally, Alice as a word has meaning: “Alice” means Truth. So, if you will, Truth has inattentively gone into free-fall from the quotidian world it has before known.)

Now most Humanities types taking their required general-education course in mathematics don't think of math that way. Instead, for them it is likely to be a series of calculations to be gotten through and perhaps a few surprising discoveries about how some basic abstract postulates can lead to necessary conclusions. By and large, these Humanities types do not think of such necessary conclusions as ushering them into a new and fantastic world. Contrastingly, the fact of Lewis Carroll and his mathematics-scientific following suggests that such mathematical people are characteristically aware of *a wonder* of mathematical discovery. We might then recommend that a first step in advanced Quantitative Literacy is to somehow convince students that they are falling into a new world and that the fall itself can be fun and funny and fantastic—and also wonderful.

A Digression into Stages of Literacy

Please notice for this discussion of Quantitative Literacy that there are steps or stages in any form of literacy. Literacy, like humor, is not truly singular but instead is a set of processes. Human beings often undertake steps of literacy in strangely idiosyncratic orders. Some of those processes may take only moments, others years, and others may never be truly completed. The completed processes are all stages in literacy, but so are the incomplete processes.

For example, in English-language literacy, an early stage is recognizing that a word is composed of several sounds and that these sounds are represented by letters. For many, pre-school instruction is enough to start this process going. For others, this process may be painfully slow. Auditory dyslexia, which inhibits the ability to sense several sounds within a word, can inhibit the whole literacy process at this point.

Such inhibition need not, however, slow other literacy processes. For example, at a much “higher” literacy level, one typically learns that what is written is not all that is communicated. There is a great deal that can be communicated “between the lines.” At an even higher level, one can distinguish between what a writer meant to communicate between the lines and what a reader can infer that a writer never meant to imply between the lines. As a professor of English, I know advance on these higher stages is possible in the presence of inhibited lower stages because I have one or more colleagues to cite as examples, who have significant auditory dyslexia problems in their backgrounds. Nevertheless, they operate at highly advanced literacy levels at least in part because they are so expert in these higher levels of literacy.

So literacy isn’t a singular something, a switch that one turns on, and that’s it. Neither is mathematical understanding, mathematical literacy, or numeracy—however defined—truly a single switch that needs turning on.

Further Carrollian Stages of (Highly Advanced) Numeracy

Within this general understanding of literacy and of mathematical literacy in particular, we have just suggested that for collegiate general education of non-STEM majors, one of the typically early processes, one of the early stages in advanced Quantitative Literacy following Lewis Carroll may very well be learning that mathematics as a mindset is a falling into a new world, a falling which if taken with Alice’s nonchalance can in itself be fun and funny and fantastic—and wonderful.

We can go a little further with Alice. She brushes herself off at the bottom of her fall and enters a narrow tunnel with numerous doors on either side, all of them locked. She finally finds a small slit in one of the doors that is opened by a golden key. Through the slit, she sees a delightful garden which she yearns to enter. Unfortunately, her body is much too big. Twelve times in the book (Carroll 1990, Gardner note 17), she is led to eating or drinking something that alters her size. We can note that size humor is thus disproportionately central to the design of *Alice in Wonderland*. Prof. Michael Catalano has suggested in private correspondence to the author that these deformations are, from a mathematical perspective, likely to be related to perception of order-of-magnitude problems.

But at a deeper (!) level, if Alice (Truth) has free-fallen into mathematics, what is the narrow tunnel? Perhaps it is precisely the Humanities mentality I just discussed. It is a long, narrow road to right answers at the end of computations. There are doors on the sides that seem like they might be more pleasant continuations, but they are all locked. One of the doors more or less accidentally allows a miniature vision of a delightful garden. However, the long hall stretches out infinitely ahead, suggesting calculations which are theoretically infinite in number but which always lead, at least potentially, to a new round of calculations.

Perhaps someone with true advanced Quantitative Literacy has not only seen the garden but has also somehow readjusted the self to be able to enter the garden.

If so, Alice for all her growing and shrinking seems infinitely delayed in getting to the idealistic goal. In that light, the humor of *Alice in Wonderland* is a fairly grim humor of delay and frustration along with size humor where constant deformation is the surprising requirement for any movement beyond present immobilization. (Consider how Alice manages to get herself into Rabbit's house by having deformed herself smaller but then incautiously deforms herself larger until she can't entirely fit in the house nor escape it without yet a further deformation.)

If Carroll and mathematicians share some such apprehension of how they moved through early stages of mathematical inquisitiveness, then perhaps if we are to move beyond Innumeracy (Paulos 1988, 1991, 1998) as a society, we need more of the population to get the picture and accept some of the frustrations. And it would probably help if we could suggest that they needed not only to get the picture but to get the humor of the frustrated situation. Alice is typically frustrated, but her humor, her mindset, is essentially respectful, inquisitive, purposeful, and adventurous. That allows her many humorous moments perhaps typified by her short introduction to the Dodo bird (Dodo was Charles Dodgson's nickname for himself; see Carroll 1990, Gardner note 27). In *Wonderland*, with the proper mental attitude, in the midst of frustrations, one comes across the strangest creatures entirely extinct in the bright ordinary surface world from which one fell!

A Final Numeracy Step in Wonderland

Let us then conclude consideration of *Wonderland* with a reconsideration of the idea that *Alice in Wonderland* was written up first as a memorial of an extemporaneous tale. It was later taken up, refurbished and published. Quite clearly, Lewis Carroll was intrigued by what he had already written and moreover enjoyed revising and refurbishing it. In fact he kept on doing so, adding an illustrator here and a stage production there for decades.

He was, then, at play. I'd suggest he was at play in a corner of the garden. Charles Dodgson had been in the garden as a mathematician. Now as Lewis Carroll, he was at play in his own fantasy-expounding corner thereof. And that means that the humor of *Alice in Wonderland* deeply appreciated is a humor of playfulness and of child-like innocence, in which Dodgson believed at least as thoroughly as he believed in mathematics itself.

Then to the sense of abysmal Intellectual humor, humor of frustration, of deformation and size, we can add a humor of playfulness alongside childish innocence and nonchalance. If *Alice in Wonderland* is nonsense, it is at least noteworthy that the humor of Alice is extraordinarily complex, definable, and

stable throughout. The presence of such persistent humor is its own strong argument that the nonsense cannot be nearly so hit-and-miss as it seems.

Perhaps this quality of being at play too should be indicated to unenthusiastic students as pre-requisite to full Quantitative Literacy. How early, then, can we introduce such a humorously unstudied, playful perspective? Again, individual human beings undergo processes of greater literacy at their own speeds and in their own time. Force-feeding a time schedule is probably much more a way of short-circuiting literacy processes than of accomplishing them. But is there an *appropriate* time at which mathematics instructors should try to indicate the playfulness of high-literacy mathematical experience? Carroll seemed himself to believe in introducing such concepts early and to believe that small children were the true heirs of such a mathematical literacy.

A Short List of Wonderland Stages of Numeracy (All of Them, High-level Numeracy Stages)

In short, then, *Alice in Wonderland* seems to point to central elements of what could be called mathematical intelligence but could also be considered as aspects of Quantitative Literacy to be inculcated if not directly taught, many of them to be elicited not just from a mathematical intelligentsia but also to some degree from non-STEM students who have become preliminarily acquainted with higher mathematics in a general-education context. These elements include:

- A sense of mathematics as a second world or parallel universe that one should allow oneself to fall into.
- A sense of mathematics as more than a narrow corridor between locked doors leading only to more and more abstruse calculations.
- A sense of the Beautiful Garden to which those versed in mathematics can attest (possibly with some parallel assertion that the Beautiful Garden has a certain riskiness about it that corresponds to the most memorable phrase in *Alice*, “Off with their heads!”)
- A sense of the mathematician conforming to the demands of the mathematical world in what are likely to seem discomfiting deformations or reformations of mind.
- A sense of frustration as a normal mathematical state to be met with an Alice-like composure, even complaisance, and an expectation that in an exciting world, something newly exciting is likely to happen.
- A sense of the accomplished mathematician as ultimately at play rather than at work—or at least *sometimes* at play interspersed with attempts to bear down, to think, and to calculate through to a conclusion.

If these elements are indeed aspects of Quantitative Literacy and indeed aspects of the journey out of Paulos' Innumeracy, then there is much to discuss about how they are to be inculcated not just for math majors or STEM personnel but for the general college-educated population. If some of my formulations are not really inherent in Quantitative Literacy, that too needs discussion not in terms of a literary analysis of Lewis Carroll by literature faculty but in terms of an earnest discussion among peers within the STEM disciplines.

Certainly these are enough of an agenda to draw from Lewis Carroll. But the fact is that he went on to a quite different sequel in *Through a Looking Glass*. *Through a Looking Glass* has its own strong intimations of other aspects of Quantitative Literacy.

A New Metaphor for Numeracy Advance: The Looking Glass

If *Alice* concerns itself at a deep symbolic level with the Wonder and Wonderland of mathematics, a congruent reading of *Through a Looking Glass* must find it deeply concerned with the Looking Glass character of mathematics and mathematical insight.

Before pursuing that lead, however, it is well to remember that *Through a Looking Glass* appears at every level of analysis to have been a carefully designed and crafted enterprise, the backbone of which is an analogy to the game of chess. Alice walks through the looking glass in Chapter I, finds herself leaving by the backdoor and encountering the Red Queen in the beautiful garden. Thereafter, the book proceeds by analogy to a chess problem which Carroll has proposed for advancing a white pawn to become a queen. Alice becomes that white pawn on a chess board composed of fields divided by brooks into chess "squares." Alice takes the normal opening double-jump of a pawn's first move by going on a passenger train.

Seemingly accidentally, she eventually encounters Tweedledum and Tweedledee, who point out the Red King sleeping in the forest behind them. They manage to insinuate that Alice may be only a figment of the king's dream, and thus subject to annihilation whenever the king awakes from his slumbers. Eventually, Alice reaches Square 8 and is promoted to being a Queen (a crown suddenly appears upon her head). And finally she is provoked by the Red Queen into shaking her ("capturing the Queen"), which shaking turns out to checkmate the Red King (that is, he is released from his slumbers—without annihilating Alice).

While the chess analogy provides the backbone and what little plot connection exists in the work, *Through a Looking Glass* is probably best remembered for the image of walking through a mirror, for incredible characters

of the behind-the-mirror world like Humpty Dumpty and the Jabberwocky, and for seemingly addled nonsense poetry that somehow is often very memorable.

Returning, then, to the simplest level, the Looking Glass analogy pointedly indicates that there is a fundamental disjuncture between mathematics and the real world. This disjuncture is congruent with the interpretation of *Alice in Wonderland* as Alice falling out of the known and commonplace world of experience into the mysterious, unknown, and quite possibly foreboding inner subterranean world of mathematics.

But the Looking Glass image comes at this issue from a very different perspective, the perspective of similarity rather than of difference. Wonderland was totally *different* from the terrestrial world. The Looking Glass, contrastively, seductively suggests a door to *identical* worlds.

Perhaps it is worth noting some or all of the following as background for a mathematical reading of the Looking Glass analogy: It is interesting that dogs and cats can sometimes perceive themselves in a glass, but they soon lose interest in the phenomenon altogether. Children, on the other hand, are entranced by the person and world on the other side of the glass, and some of them maintain a life-long absorption therein. Charles L. Dodgson seems to have been one of these, perhaps particularly drawn to looking glasses because his own features were quite idiosyncratically non-symmetric (Carroll 1990, Gardner *xvi*). And for children, the enchantment originally is a sense that they are looking exactly at their own world.

It takes cognitive effort to recognize that no, the world on the other side of the glass is not identical to our own but rather has been left-to-right transposed. (It probably is beyond most children and most adults to recognize the further anomaly that the transposition is *only* left-to-right, *not* left-to-right *and* also top-to-bottom. And then there are the additional mysteries, probably uninvestigated, that the mirror has the enviable ability and necessity when we turn it sideways to continue its left-to-right transposition and also its top-to-bottom non-transposition only now on new axes.) Additionally, the world on the other side of the mirror lacks all but visual properties and thus does not have the smell of garlic or the sense of oppressive heat that may be in the room reflected in the glass. Perhaps these lacks along with the ultimate two-dimensionality of the reflected room are at stake in animals' general disinterest.

In a certain sense, this transposition is all very amusing and ultimately an investigation of the human mind. The humor of *Through a Looking Glass* will always have a relationship to this fundamental Incongruity humor, a very mathematical, left-to-right-transposition humor that typically escapes almost everyone else.

Lewis Carroll leads us to consider the Looking Glass more carefully and mathematically. Do general-education courses in mathematics lead their students

to recognize that mathematics are in a similar relationship to daily reality, both intimately tied to that reality and at the same time by fixed laws of the universe always separate from that reality? This too is one of the journeys, one of the processes by which we leave Paulos' Innumeracy and arrive at new and fuller understanding.

There is another aspect of the Looking Glass that Carroll's grand design fully demonstrates from the first chapter onward but which he doesn't explicitly comment upon until he does so indirectly half way through the book in Humpty Dumpty's exegesis of Carroll's earlier poem, "Jabberwocky." In defining the word "wabe" as in "Did gyre and gimble in the wabe," Dumpty says it means "way be—" "you know, because it goes a long way before it, and a long way behind it—."

And isn't that what we all think of a mirror, joining everything before it with everything after it within the field of sight? But note that Dumpty has been cut off in the middle of his sentence by Alice, who continues, "And a long way *beyond* [my italics] it on each side." To which Dumpty responds, "Exactly so" (Carroll 1990, 215).

Are Alice and Dumpty still metaphorically talking about a mirror? Yes, if it is Carroll's mirror. Going back to the earliest chapters, Alice entered through the Looking Glass into a room both like and unlike the real-world room she left. But ultimately, she leaves this room *through its backdoor*, beyond what we can see from the real world, eventually coming to another garden and the view of a new hill. So the glass is not just the doorway between the real-world room and the somewhat different room behind the mirror. It is also the doorway to something *beyond* the already-different room, something *beyond* what we can imagine in the real world.

Even those of us outside the STEM disciplines have some vague sense that such a world *beyond* turned out to be real in terms of Relativity Theory and Quantum Mechanics.

In some senses, this world beyond is another view of the Beautiful Garden of *Alice in Wonderland*. And the understanding that such a garden is not entirely separated from reality but instead is a separated-but-bound-to reality apprehensible through mathematics is again a process, a journey out of Innumeracy. Can mathematics inculcate that sense of wonderful possibility? If no one can think of any better way, perhaps we should start by retelling *Through a Looking Glass* to students.

Separated-But-Bound Mathematics and the General-Education Student

The journey out of Innumeracy that leads to the perception that mathematics is eternally separated from reality while being simultaneously bound to it is easily

within the grasp of general-education students. It may, in fact, be a perception they have already privately nurtured. In which case, it would be bridging the gap between mathematicians and those outside STEM disciplines to explicitly recognize the fact of this separation and thus, inherently, give credit to students who have already privately perceived it.

Contrastingly, the journey out of Innumeracy that leads to a perception that mathematics finally moves not just *behind* the mirror but *beyond* the mirror may be thoroughly beyond the direct understanding of general-education students or even beyond most majors. The fact that it is beyond their direct understanding does not, however, mean that we cannot help them undertake the process of leaving Innumeracy to some extent by *taking our word* for what profound mathematicians have found to be true. Again, retelling Lewis Carroll may be one of the more persuasive ways to encourage this process out of Innumeracy.

Additional Numeracy Projects in Through a Looking Glass

A third major aspect of mathematical humor in *Through a Looking Glass* again returns to something that even rather dull general-education students already know, but they know it backwards. Such students typically find the rigors of mathematics trying and may easily feel that logic might just as easily go somewhere. And indeed, logic can go somewhere. Both *Alice in Wonderland* and *Through a Looking Glass* teach this lesson incessantly and humorously.

Given different postulates (or premises), all reality shifts. Given enough different postulates, “Off with their heads!” is the solution to every problem. There are historical examples of societies that had enough of those different postulates and that historically, in fact, decided that “Off with their heads” was a generally acceptable solution to life’s problems. Hitler’s Germany is routinely cited in this regard. At least as far as genocide was the issue, one could as easily cite Stalinist Russia. Henry VIII’s England may be more directly referenced by Carroll.

So learning that postulates govern conclusions and also that postulates may be arbitrarily chosen (though perhaps at considerable pragmatic risk) are not just mathematical lessons. They are of universal importance for understanding human thought.

Lewis Carroll was inveterately fond of teaching the final unchallengeability of postulates. And he is a past master of teaching that lesson humorously. Three examples suffice.

In “It’s My Own Invention,” Chapter VIII of *Through a Looking Glass*, the Red and White Knight fight over control of Alice, all according to the Rules of Battle. Alice hasn’t been told the specifics of these rules, so she spends time during the battle trying to intuit what the rules must be. All of which leads to the culminating moment of conflict:

“Another Rule of Battle, that Alice had not noticed, seemed to be that they always fell on their heads; and the battle ended with their both falling off in this way, side by side.” Well, in the postulated universe, this sort of clumsiness is perhaps inevitable.

But then the text continues (Carroll 1990, 235):

When they got up again, they shook hands, and then the Red Knight mounted and galloped off.

‘It was a glorious victory, wasn’t it?’ said the White Knight, as he came up panting.

Now isn’t the White Knight purely absurd? From real-world battle postulates, of course he is. But from the rules of Carroll’s imagined world, it is the *only* congruous conclusion possible. There is a grand chess theme to the whole design of *Through a Looking Glass*: the White Knight has landed on the Red Knight’s square, so, of course, the outcome of the battle is already determined no matter how many times whoever falls on his head.² It is always a glorious and total victory. The only question is whether you get to stay on the square or whether you are removed from the square (and from the board), and that has already been answered by who landed on whose square. The chess game in *Through a Looking Glass* is not entirely the chess game known in real Victorian England, and thus, evidently, the last clause of the rule, “removed from the board,” has itself been removed from Carroll’s rules.

That’s how it is in Carroll’s game: get used to it.

The absolute dominion and authority of postulates is also at stake in what many consider an incomprehensible dialog between Alice and the White Queen. The Queen says, “it’s very good jam.” And Alice responds, evidently trying to find some common ground of understanding, “Well, I don’t want any to-day at any rate.” “You couldn’t have it if you *did* want it,” the Queen said. “The rule is, jam to-morrow and jam yesterday—but never jam to-day” (Carroll 1990,196).

Here Carroll’s interminable interest in puns is at the heart of the joke, a pun in two languages. In Latin, *iam* or *jam* means “now” either in the past or the future. But in the present, the idea of now is represented by *nunc*. Thus you can have *jam* to-morrow or yesterday, but not today. But equally at the heart of the joke is that going off onto a second, totally irrelevant meaning of words is entirely appropriate conduct and quite possibly authoritative in Carroll’s world.

Again, get used to it. He who makes the postulates makes the rules and determines the outcome of the game beforehand. The game well played is simply a playing through to these inevitable conclusions. Here we are back to Carroll’s

² In real-world chess the piece moving onto another’s square always wins. So here, the White Knight must win even if both fall on their heads.

playfulness, and perhaps it is a higher process out of Innumeracy to recognize the Queen's authority to name the game even amidst total real-world absurdity.

(It should perhaps be at least considered for a moment that many other disciplines may have adopted the mathematical unchallengeability of postulates. If so, it may be part of liberal education to consider whether the unchallengeability of postulates is always wholesome when it migrates to other disciplines.)

And as a third example, we come back to Humpty Dumpty who can define “wabe” (rhyming with “outgrabe” and thus almost certainly a single syllable; moreover, Carroll's iambic tetrameter seems to demand that “wabe” be pronounced as a single syllable) as, in fact, a combination of “way” and “be-” as the prefix in “before,” “behind,” and “beyond.”

Dumpty has already explained, “When I use a word it means just what I choose it to mean—neither more nor less” (Carroll 1990, 213). It is a masterful definition of technical vocabulary. And it really should be remembered in understanding a technical essay in literary criticism as much as in understanding a technical discussion in any of the STEM disciplines. Technical discussion needs postulates, exact postulates wherever possible. Toward that end, let all practical worldly meaning go hang. A word in technical discussion becomes a proto-postulate, and for that discussion it means “what I choose it to mean” (as arguer) “nothing more nor less.”³

In this sense, understanding the arbitrariness of technical vocabulary—not getting hung up on it, not wasting time arguing over it instead of seeing where the postulate would lead—is both a process of leaving Innumeracy for Numeracy and of leaving Illiteracy for Literacy.

(Again, going beyond mathematics, in the Humanities, one often senses that arguers understand this educated tendency all too well, and understanding it, choose biasing language as their technical terms, essentially winning a cheap trick at the expense of too-compliant, but educated audiences.)

Are Literacy and Numeracy Separate-but-Bound terms?

Perhaps, then, Numeracy and Literacy (capitalized to reflect the technical nature of its meaning in this discussion) are two sides of the same mirror.

Walk through the mirror, and perhaps we find an entirely new understanding in which the problems Humanities personnel address as Illiteracy and the problems STEM personnel address as Innumeracy have a great deal in common and a great deal to be mastered together rather than separately. And the great problem then for Humanities and STEM personnel is the problem of leading

³ Perhaps “numeracy” vs. “quantitative literacy” vs. “quantitative reasoning” is a case in point of technical language? (e.g., Vacher 2014, Karaali et al. 2016).

students out of their natural world to another which can seem distorted and fractured, even absurd and menacing, but is ultimately beautiful and powerful taken back into the natural world.

I am a member of a Humanities faculty, not of a STEM discipline. Do I have everything right about the nature of Numeracy as processes away from Innumeracy as explicated by Lewis Carroll? I think that highly improbable. Have I covered all of Lewis Carroll's mathematical interests and their humorous manifestations in *Alice in Wonderland* and *Through a Looking Glass*? Certainly not.

But hopefully, I have postulated, following John Allen Paulos' leads, that:

- The world is facing an Innumeracy crisis because *our world has become so numerate* even if we human beings haven't.⁴
- Mathematics does have its own—admittedly quirky—humor which can be considered either an occupational hazard or an occupational blessing. As much as possible, we should try to understand that humor to make it as much an occupational blessing as possible.⁵
- Mathematicians do see the world differently, which is related to their humor and related to their professional processes.⁶

And I have further postulated that there is a reading of the Lewis Carroll classics that speaks both humorously and profoundly to what it means to be numerate at advanced levels.

The mistakes from there are mistakes in my understanding of mathematics or of my literary judgment. Assume both.

The question, however, is not what were my mistakes but rather what remains after mistakes have been erased (in other words, Carroll's Cheshire Cat, the smile that remains when the rest has disappeared). Has Lewis Carroll spoken to issues of Numeracy and particularly to seeing Numeracy as processes by which the child leaves the natural world and natural Innumeracy for fuller understanding? If so, do we as modern society need to do more or do more *effectively* to help the young move through Numeracy processes? In particular, can we help them move toward fuller mathematical understanding useful back in their own mundane worlds?

⁴ See Paulos (1988), *Innumeracy*, throughout.

⁵ See Paulos (1980), *Mathematics and Humor*, and "A Mathematical Accent" in Paulos (1991), *Beyond Numeracy: Ruminations of a Numbers Man*. For more on Paulos' *Mathematics and Humor*, see Grawe (2015).

⁶ See Paulos (1980), *Mathematics and Humor*, throughout, as well as much in Paulos (1995), *A Mathematician Reads the Newspaper*. For more on Paulos' *Mathematics and Humor*, see Grawe (2015) and Grawe (2016).

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